## **Tidal Constituent Quality and Prediction Error Budgets**

## Anne Ballantyne, Canada

#### **SUMMARY**

Accurate water level information is very important for proper sounding reduction. The increasing need for the collection of hydrographic data in the Arctic and other remote areas has highlighted the difficulty in the collection of observed water levels in such conditions. This paper describes the findings of a Canadian Hydrographic Service (CHS) Pacific Region study examining how the variance between predicted and observed water levels is affected by the record length of observations used to compute harmonic constituents. The results of this study should aid decision making when weighing the implications of collecting observed water levels; including vessels costs, time restraints and inhospitable environments; against using predicted water levels by giving a measure of estimated uncertainty. The lengths of observed records considered were 1 year, 6 months, and 29 days. The results for 10 day observation lengths were also studied for two locations. Nineteen-year predictions from each harmonic constituent set were subtracted from 19 years of observed data and standard deviations were computed from these differences. The paper describes full study results for four locations: Resolute, Nunavat; Point Atkinson, British Columbia; Churchill, Manitoba; and Yarmouth, Nova Scotia. Results for Victoria, British Columbia do not include 29 day results. The uncertainty estimates at 95 % confidence level are calculated to match the definition in the IHO S44 standard. Some of the results run counter to long held assumptions.

**Key words**: Prediction, uncertainty estimate, water level, tides, sounding reduction, harmonic constituent

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## 1. INTRODUCTION

Collecting the observed water levels, needed to perform proper sounding reduction, is difficult in the Arctic and other remote locations. Until a continuous vertical datum model is available the hydrographer is often faced with weighing the implications of collecting observed water levels, with its vessels costs, time restraints and inhospitable environments, against using predictions. Without an objective method to assess the uncertainty of predictions, the hydrographer is left with no means to make this decision even with the advice of an experienced tidal officer.

The paper describes a study carried out to find estimates of uncertainty for predicted water levels based on the category of harmonic constituent sets used to produce them.

The Canadian Hydrographic Service (CHS) harmonic constituent sets can be placed in the following categories based on length of observations: multiple year vector averaged, single year, 6 month to 1 year, 29 day to 6 month, sub-month, and 29 day 10 constituent. The first category is based on the work of W.R. Crawford as described in Crawford (1995). Up to 19 individual years of harmonic analysis were calculated, the individual constituents were then vector average after the removal of anomalous years. The next four categories are constituents produced by the Foreman analysis described in Foreman (1977). The Foreman analysis allows the constituents that normally could not be separated due to the length of the record to be inferred from an input relationship. The last category, 29 day 10 constituents, was produced using the historic semi-graphical method. This method is described in Suthons (1959). The 10 constituents found in Canadian constituent set produced by the semi-graphical method are Z0, O1, P1, K1, N2, M2, S2, K2, M4 and MS4.

## 2. METHOD

Predicted water levels were compared to observed water levels in this study. Due to effects of barometric pressure and winds complete agreement was not expected. To allow the inclusion of the full 18.6 year nodal modulation of the lunar constituents, the difference between 19 years of observed water levels and the same 19 years of predictions were calculated. The standard deviation of the differences was recorded. To create data sets to match the constituent categories the 19 years of observed data were divided into segments. The category of 6 months to 1 year was represented by successive sections of 184 days starting at the beginning of each month and the 1 month to 6 months by segments of 29 days. Once the 29 day harmonic analysis was completed a new constituent set was created from the set of 33 constituents including only those constituents produced by the semi-graphical method. Since the 29 day 10 constituent set included the constituents P1 and K2 which cannot be separated at 29 days, the 29 harmonic analysis had to infer P1 from K1 and K2 from S2. This was not difficult because the necessary factors are well known for the long term permanent water level stations studied. Some 1 month to 6 month day constituents used by CHS do not include the inferred constituents therefore another category, the 29 day harmonic analysis that is not inferred, was added to the study. Predictions were produced using the Foreman prediction method described in Foreman (1977) for all categories of constituent sets for the same time span and time intervals as the observed water levels. The standard deviation of each category of differences that contained more than one prediction was averaged. The uncertainty estimate at the 95% confidence level was calculated by multiplying the resulting standard deviation value by 1.96 to match the 95% confidence level in the IHO S44 standard for 1 dimensional quantities. All subsequent references to uncertainty will be at the 95% confidence level unless otherwise stated.

A full 19-year set of water levels with no data missing would contain 228 sets of 29 day observations and 222 sets of 6 months observations, if each observation set started at the beginning of each month.

### 3. RESULTS

## 3.1 Mixed semi-diurnal tidal regimes

### 3.1.1 Resolute, Nunavat

The data archive for Resolute shows a 19-year record from 1958 to 1976. There is missing data of almost 365 days starting in November 1960 and ending in October 1961, and 6 other 30 day sections of missing data exist in the remaining data record. Since data collection started in 1957 and ended in 1977, there is no way to have a continuous 19 years that does not include the missing year. The official constituents are vector averaged and give an uncertainty of 22.7 cm. Contrary to expectation the semi-graphical constituent set has an uncertainty which is 5.9 cm less than the 29 day not inferred set and 1.6 cm less that the 29 day inferred set it was created from and 1.4 cm less than the 6-month set. The 10 day analysis with its 43.6 cm uncertainty was included because some arctic constituents set are based on observed data sets of approximately this length.

The following table lists the constituent types and the uncertainty estimates in the order from least uncertainty to most.

Resolute	Uncertainty estimate (m) [95% confidence]	Number of standard deviation values averaged
Vector Averaged	0.227	n/a
1 year	0.268	19
29 days, 10 constituents to approximate the historic semi-graphical method	0.277	205
6 months	0.291	174
29 days with P1, K2 and N2 inferred	0.293	206
29 days with no inference	0.336	206
10 days with P1 and N2 inferred	0.436	210

## 3.1.2 Point Atkinson, British Columbia

There are observed water levels available for Point Atkinson from 1914 to the present. The 19-year record used in the study is from 1991 through to the end of 2009. This record unfortunately contains a 4 month section of missing data between September 1997 and December 1997 and one other occurrence of missing data of more than a month. The present vector averaged constituents set (1975 to 1994) gives an uncertainty estimate of 28 cm. Once more the semi-graphical constituent set has an uncertainty estimate that is less than the full 29 day inferred set. In this case, inferring the constituents reduces the uncertainty estimate by 28.4 cm.

The following table lists the constituent types and the uncertainty estimates in order from least uncertainty to most.

Point Atkinson	Uncertainty estimate (m) [95% confidence]	Number of standard deviation values averaged
Vector Averaged	0.280	n/a
1 year	0.304	19
6 months	0.329	218
29 days, 10 constituents to approximate the historic semi-graphical method	0.382	213
29 days with P1, K2 and N2 inferred	0.385	222
29 days with no inference	0.669	222

## 3.2 Semi-diurnal tidal regimes

### 3.2.1 Churchill, Manitoba

Churchill has been a permanent station since 1940. The water levels are a combination of river discharge and tides. The most recent 19-years, 1993 through to the end of 2011, were chosen for this study. The uncertainty estimate for the vector averaged constituents is 35.1 cm. Once more the semi-graphical constituent set has a smaller uncertainty estimate than the 6-month and the 29-day inferred and the 29-day not inferred constituents. The 10 day constituents have an uncertainty of 84.9 cm. Inference decreased the uncertainty by 8.0 cm.

The following table lists the constituent types and the uncertainty estimates in order from least uncertainty to most.

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Churchill	Uncertainty estimate (m) [95% confidence]	Number of standard deviation values averaged
Vector Averaged	0.351	n/a
1 year	0.419	19
29 days, 10 constituents to approximate the historic semi-graphical method	0.468	228
29 days with P1, K2 and N2 inferred	0.475	228
6 months	0.477	223
29 days with no inference	0.555	228
10 days with P1 and N2 inferred	0.849	228

## 3.2.2 Yarmouth, Nova Scotia

Yarmouth has been a permanent station since at least 1965. The 19-years used for the study were from 1991 to 2009. The observed data has many sections of missing data, including 16 individual months that do not include enough water levels for a 29-day analysis. The vector averaged predictions have an uncertainty estimate of 25.5 cm. In this case, the uncertainty of the semi-graphical constituent set falls in between the values for the 29-day inferred and the 29-day not inferred values. The 29-day inferred set has an uncertainty estimate 7.3 cm less than the not inferred set.

The following table lists the constituent types and the uncertainty estimates in order from least uncertainty to most.

Yarmouth	Uncertainty estimate (m) [95% confidence]	Number of standard deviation values averaged
Vector Averaged	0.255	n/a
1 year	0.274	19
6 months	0.312	223
29 days with P1, K2 and N2 inferred	0.327	212
29 days 10 constituents to approximate the historic semi-graphical method	0.332	212
29 days with no inference	0.400	212

## 3.3 Mixed mainly diurnal tidal regime

## 3.3.1 <u>Victoria, British Columbia</u>

There has been a gauge at Victoria Harbour since 1899. The selected time for this study was from 1991 thru 2009. There are no sections of missing data greater than one month. The uncertainty estimate for the semi-graphical set fall in between the 29-day inferred and 29-day not inferred constituent sets. The 29-day not inferred constituent set has an uncertainty estimate that is 18.3 cm greater than the 29-day inferred set.

The following table lists the constituent types and the uncertainty estimates in order from least uncertainty to most.

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Victoria	Uncertainty estimate (m) [95% confidence]	Number of standard deviation values averaged
Vector Averaged	0.257	n/a
1 year	0.287	19
6 months	0.310	222
29 days with P1, K2 and N2 inferred	0.324	226
29 days 10 constituents to approximate the historic semi-graphical method	0.326	226
29 days with no inference	0.517	226

## 3.4 Diurnal tidal regimes

According to Fig. 22 in Forrester 1983 there are two small areas of diurnal tides in Canadian Waters. Neither of these have any long continuous observed water levels so no calculation were possible for this study.

#### 4. CONCLUSIONS

The vector averaged constituent sets produce the smallest uncertainty estimates of all the constituent categories. Four locations have uncertainty estimates less than 30 cm but more than 26 cm with the values for Churchill higher due in large part to the effects river discharge. Only Resolute has an uncertainty estimate less than 25 cm and only for the vector averaged constituent set

If the study results are extrapolated to all the constituent sets in CHS holdings then the order of reliability of constituents sets, from most to least, is as follows: vector averaged, 1 year, 6 months to 1 year, 29 days to 6 months with inference along with the semi-graphical, followed by 29 days to 6 months without inference, and less than 29 days. The uncertainty estimate of the 10 day constituent set was 51 cm larger than the vector averaged for Churchill and 21 cm larger for Resolute. It is interesting to note that the uncertainty estimates for 10 days are 22% of range for Resolute and 18 % for Churchill.

When this study began the long held assumption was that the more constituents in a set the better the predictions it would produce. The results of this study clearly show that the assumption was wrong. The uncertainty estimates for the semi-graphical method sets, with only 10 constituents are consistently less than for the 30 constituent 29-day not inferred sets. In fact in some cases, the semi-graphical method sets have an uncertainty estimate less than the 33 constituent 6-month constituent sets, not to mention the 33 constituent 29-day inferred sets.

This study shows that few existing constituent sets can be used to replace observed water levels with acceptable uncertainty estimates. The IHO S44 standard states that for areas where under keel clearance is critical, the total vertical uncertainties that do not vary with depth cannot exceed .25 m. This means that of the locations studied only the predictions from vector averaged constituent set for Resolute could possibly considered for use for such a survey and only if there were no vertical uncertainties.

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## **BIOGRAPHICAL NOTES**

Anne Ballantyne has been working in the tides and current field within CHS for the past 28 years. During that time she has been key in developing custom software tools for Tide and Current Table Production and tools for tide and current data processing and analysis (TCWL Tools), for extracting and converting tidal constituents from the Foreman tidal models and enhancing TCWL tools to enable batch processing of tidal model constituents (predictions, target datum calculations). Anne has also participated in Tide and Current Table production, processing of tide and current data (pressure gauge and current meter) as well as providing software support for data collected in the Permanent Water Level Network. She currently maintains the Fraser River Model and is responsible for all National TCWL Tools applications. Anne has worked with Dr. Mike Foreman, DFO physical oceanographer (circulation and flow modelling, tidal analysis, data assimilation) on tidal modelling and Dr. William Crawford DFO physical oceanographer (currents, tides and ocean mixing).

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