The Canadian Hydrographic Continuous Vertical Datum: Methodology and Accuracy

Catherine ROBIN*, Andre GODIN, Phillip MACAULAY, Bodo DE LANGE BOOM, Denis LEFAIVRE , Terese HERRON, Denny SINNOTT, Anne BALLANTYNE, Louis MALTAIS & Marc VÉRONNEAU, Canada

SUMMARY

The Canadian Hydrographic Service (CHS) has developed a program to build a separation model (SEP) which seamlessly maps the height of chart datum (CD) above the GRS80 ellipsoid in the NAD83(CSRS) reference frame over all Canadian waters, making this product compatible with spatial positioning technique (e.g., GPS). The Canadian Geodetic Survey is providing important support to the completion of this program. An accurate separation model will improve on existing local SEP's by incorporating data from ocean models and/or satellite altimetry. Unlike similar programs in other countries, the Canadian seamless datum is managed using a top-down approach, with 4 regional models covering all Canadian waters, and is constrained by relatively few observations due to the large size and inaccessibility of the territory. In this paper, we demonstrate a procedure for calculating a seamless CD, and examine preliminary results for the Gulf of St. Lawrence.

Key words: seamless chart datum, tidal modeling, sea level, height reference

1. INTRODUCTION

The Canadian Hydrographic Service (CHS) has developed a program to build a separation model (SEP) which seamlessly maps the height of chart datum (CD) above the GRS80 ellipsoid in the NAD83(CSRS) reference frame over all Canadian waters, making this product compatible with spatial positioning technique (e.g., GPS). It will improve on existing local SEP's by incorporating data from ocean models and/or satellite altimetry. Unlike similar programs in other countries, the Canadian seamless datum is managed using a top-down approach, with 4 regional models covering all Canadian waters, and is constrained by relatively few observations due to the large size and inaccessibility of the territory (see Lefaivre et al. (2010)). A brief history of and the motivation for the development of the project is available in de Lange Boom et al (2012), Lefaivre et al. (2010), and O'Reilly et al. (1996). In this paper, we demonstrate a procedure for calculating a seamless CD, and examine preliminary results for the Gulf of St. Lawrence. Other datums may be calculated in the final version of the Canadian Hydrographic Continuous Vertical Datum in much the same as we present here; we limit our discussion to the SEP for CD and its target datum Lower Low Water Large Tide (LLWLT).

2. METHODOLOGY

2.1 General Overview

There are a number of regional SEP's currently in operation, generally produced by interpolation (kriging) of GPS-observed CD-NAD83 separations at a more or less dense array of tide gauges (e.g. Lefaivre et al. (2010)). This type of model does not take into account changes in tidal regime between gauges or offshore and thus may be inaccurate over large bodies of water, in areas of complex tides, and where gauges are sparsely distributed. We seek to improve the SEP between gauges and offshore in a physically meaningful way. For the Gulf of St. Lawrence, this is done using data from the G5 ocean circulation model to estimate the height of mean sea level (MSL) above LLWLT, and Natural Resources Canada's most recent geoid model CGG2010 to link MSL to the ellipsoid.

The procedure for integrating observations, the geoid, and an ocean circulation model can be divided broadly in two parts, as shown in Figure 1. The notation used in this figure and the discussion that follows is simplified from the standard notation used in geodesy (NRCan, 2012), with the exception of Sea Surface Topography (SST) which here shall be called Dynamic Ocean Topography (DOT) to avoid confusion with sea surface temperature, as per de Lange Boom (2012).

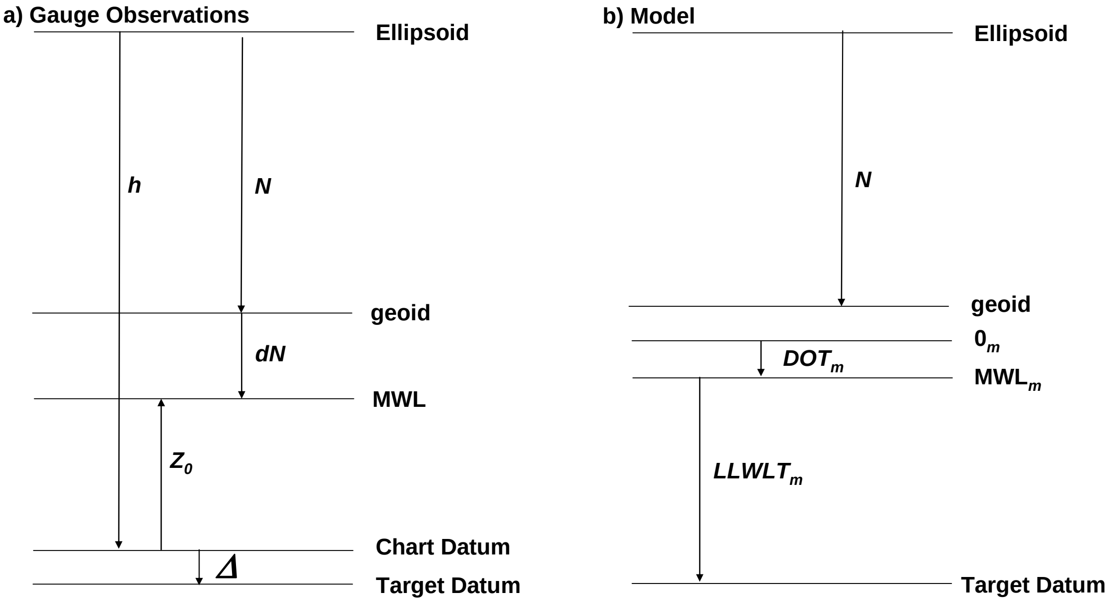


***Figure 1*. This figure shows the information available at gauges (a) and at nodes of the ocean circulation model (b) which is pertinent to the calculation of the SEP. We use the convention that +ve is upward and –ve is downward. Other tidal datums are available at most gauges and from the model, some of which are used to validate the oceanic model and will be defined in the text. Note that in some papers *DOT* is referred to as the z_0 of the model; this terminology is rather confusing in this context and is not used here.**

Figure 1a illustrates the range of observed data available at tide gauges used in the SEP calculation: h is the separation between CD and the ellipsoid; Z_0 is the separation between MWL and CD; Δ is the difference between CD and its target datum $LLWLT_o$ (=-$Z_0+\Delta$); N is the separation between the ellipsoid and the geoid; and dN is the difference between the geoid and local mean water level (MWL). h is measured in situ by GPS, levelled from a local benchmark; MWL is the average of water level observations at the gauge; N is from NRCan's geoid CGG2010 (calculated at each lat/long using GPS-H (NRCan, 2011)); dN is calculated below; and Δ is calculated from a 19-year prediction of water levels using constituents based on a harmonic analysis of available observations, using the method of Foreman & Godin (Foreman, 1977; Godin, 1972). The difference between CD and the target datum (Δ) arises from limited observations or poor tidal analysis tools available at the time CD was established, from hydrographic operational procedures, or from changes in relative water level due to absolute sea level rise and/or crustal motion. For the gauges selected for this modeling exercise, Δ is on average ~ 7 ± 13 cm (mean and standard deviation). As CD values are updated, Δ will decrease. Nevertheless, all official CD values whose ellipsoid height has been established with GPS will be honoured in the SEP. Note that any one tide gauge may have any or all of these observations recorded; some may have only Z_0, others will have Z_0 and h but not Δ, and yet others will have Z_0 and Δ but not h.

Figure 1b shows the relevant data available at each node of the G5 oceanic circulation model: N is the separation between the ellipsoid and CGG2010 as in Figure 1a; $LLWLT_m$ is the separation between the target datum and MWL; and DOT_m is the dynamic ocean topography of the model, i.e. the deviation between mean water level once the model has reached statistical steady-state and the equilibrium water level imposed at time $t=0$ which serves as a zero height reference for the model (0_m). Thus 0_m represents an equipotential surface whose relationship to the geoid is unknown.

As the zero of the G5 model is a free surface, we fix it to the equipotential surface best representing MSL in our study area by adding it at every node to $N + \langle dN \rangle$, the average dN for gauges in the model domain. A 'model SEP', representing the separation between $LLWLT_m$ and the ellipsoid, is defined at each node of the model by

$$SEP_m = N + \langle dN \rangle + DOT_m + LLWLT_m \tag{1}$$

while at each tide gauge:

$$h + \Delta + \varepsilon = SEP_m = N + \langle dN \rangle + DOT_m + LLWLT_m \tag{2}$$

where ε is the discrepancy between the model SEP and observations. ε includes errors such as levelling errors, benchmark instabilities, errors in the geoid, limitations of the ocean circulation model, and errors induced by the SEP calculation procedure itself. In other words, it includes both observational biases and modeling biases. It is desirable to separate them where possible in order to estimate the accuracy in the hybrid SEP, especially as observations and models are independently updated and improved. Details of how these biases or errors are tracked and propagated through the hybrid SEP will be available in the future. For the moment, ε is an indicator of the model SEP's ability to predict observed LLWLT-ellipsoid separation values. In addition, it is used to optimize the hybrid SEP by eliminating the average observation-model bias over the study area, after verification that the differences are distributed in a relatively random fashion.

In the final stages of the calculation, the model SEP is warped to honour established values of CD in the vicinity of gauges, creating a hybrid SEP which integrates observations and the model SEP. This is done using a procedure which allows the model to shape the datum in an along-shore direction between gauges, and to transition to the pure model in a smooth and controlled fashion in the off-shore direction.

2.2 The G5 Ocean Circulation Model

The G5 ocean circulation model uses an implicit finite difference scheme developed by Backhaus (1983, 1985), modified by Stronach et al. (1993), calibrated over the Gulf of St. Lawrence by Saucier et al. (2003), and applied to hindcast for 1997 by Senneville et al. (2011). The year 1997 was chosen from the available runs 1997 to 2009 because it was a typical year for freshwater run off from the St. Lawrence River basin. In addition to fresh water run off, the model is forced by atmospheric winds, by tides imposed at the straights of Cabot and Belle-Isle calculated using 27 tidal harmonic constituents from the nearest tide stations, and is coupled to a dynamic sea ice model. Water levels are extracted at 60 minute intervals for a period of 1 year on each node of the 5-km square grid. Tidal analysis is performed on each time series, and the resulting constituents used to predict 19 years of water levels, from which is calculated LAT, HAT, HHWLT and LLWLT. The model’s Dynamic Ocean Topography (DOT) is also computed by the tidal analysis, and is the result of fresh water flow through the domain as well as forcing by tides and winds.

3. RESULTS

The Gulf SEP discussed here covers most of the model domain of the G5 model, truncated to avoid overlap with the St. Lawrence SEP, as shown in Figure 2. All gauges within 15 km of the model domain are retained for use in the calculation. Model results are validated and/or adjusted by comparing gauge observations to model data at the closest node in the model domain; some error will be introduced by this interpolation. Except for this necessary horizontal transfer, however, no other interpolations off the grid are used. All data is retrieved and calculations performed directly on the nodes of the G5 model.

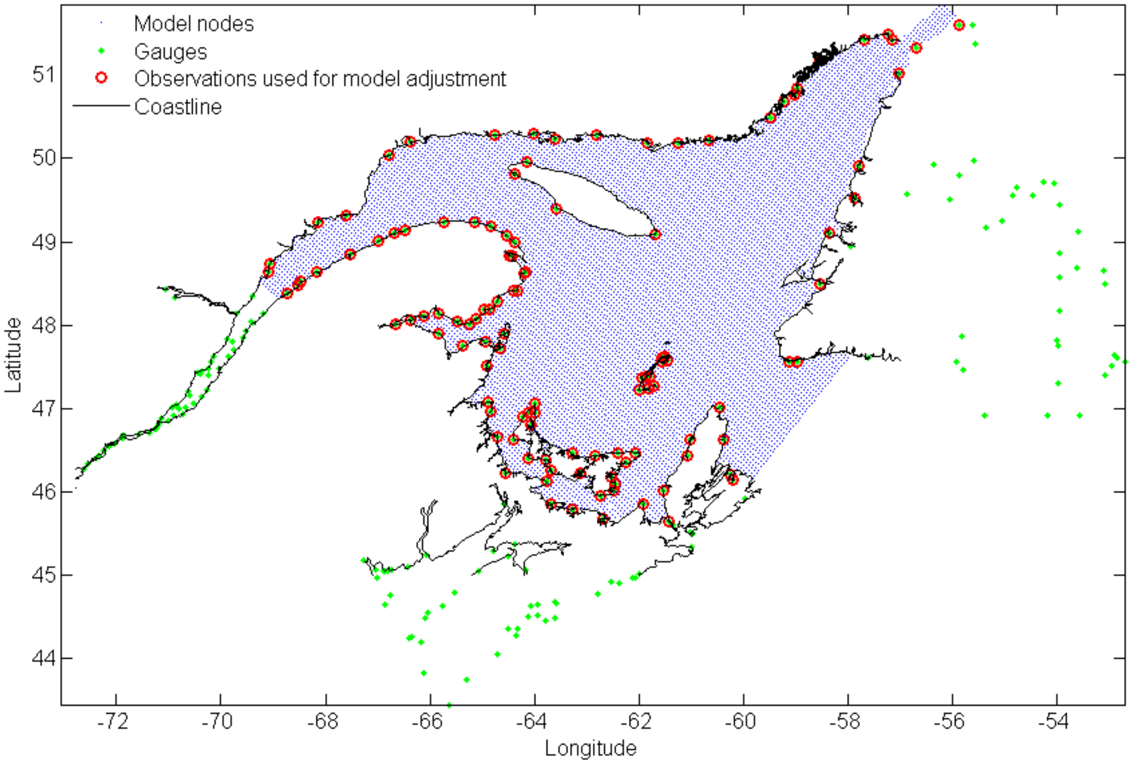


***Figure 2*. Gulf SEP model domain and selected tide gauges.**

3.1 Tidal Datum Adjustment

We begin by comparing observed and modeled datums. The correlation between observations and G5 datums for LLWLT and its high water equivalent HHWLT is good (Figure 3a,b), with correlation coefficients of ~ 0.96 and ~ 0.98 respectively. However, the model has some difficulty capturing the tidal asymmetry (the difference in amplitudes of the high and low datums relative to MSL), measured here as $\frac{HHWLT}{LLWLT}$. Indeed, Figure 3c shows that the correlation coefficient for tidal asymmetry is only ~0.80. Furthermore, the spatial distribution of the observation to model differences in LLWLT displays a SW-NE trend, shown in Figure 3d, with the model overestimating the amplitude of LLWLT by a few percent on average.

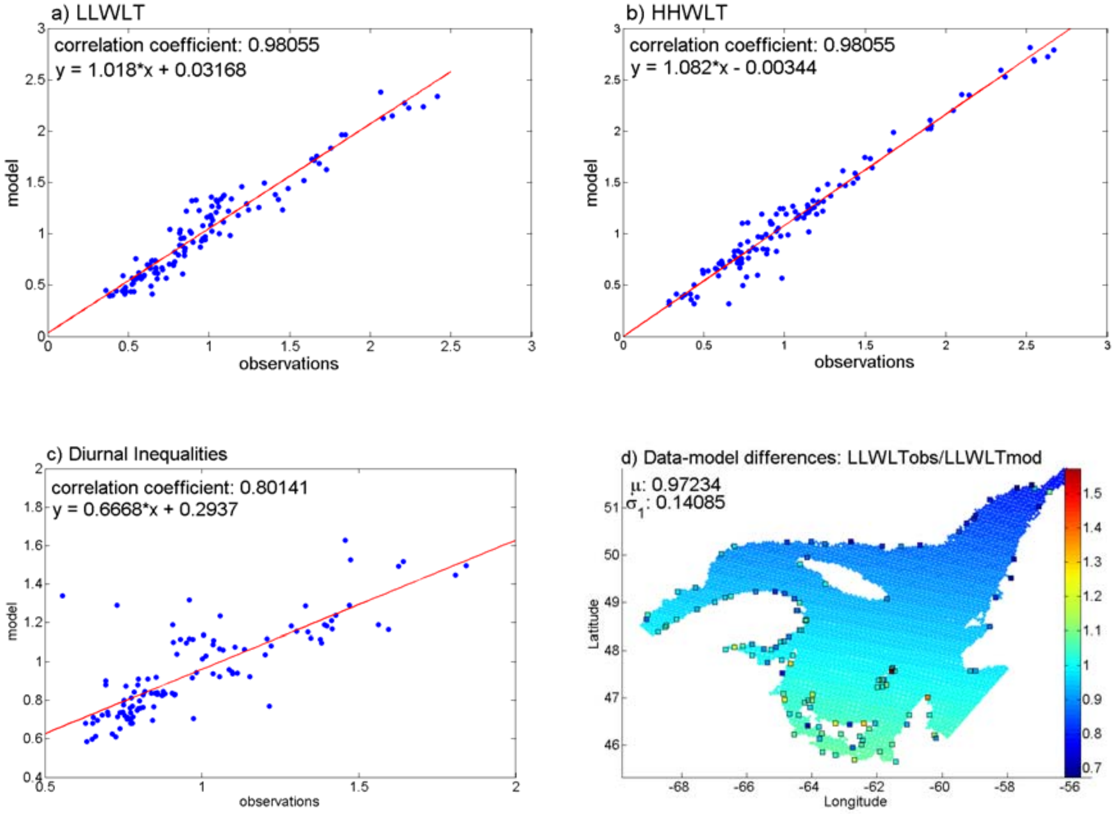
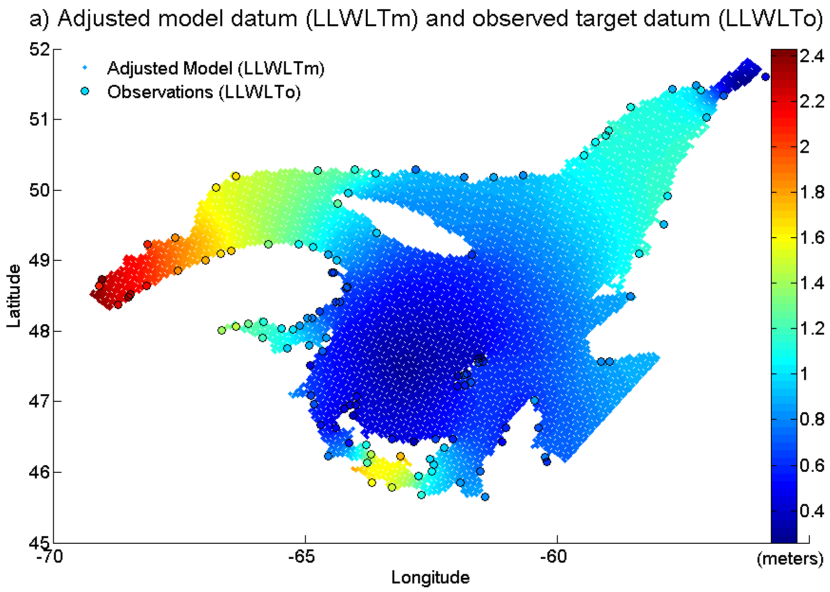


Figure 3. **Correlation of model and observed datums. Note that in this figure LLWLT is plotted as a positive value.**

Therefore, instead of constraining the model's tidal range by the observed tidal range ($LLWLT_m$+$HHWLT_m$, which the G5 model reproduces better than LLWLT), we adjust $LLWLT_m$ directly using ratios extracted from observations as a plane through $LLWLT_o/LLWLT_m$. The ratios vary from ~0.78 in the Straight of Belle Isle to ~ 1.1 in the Northumberland Straight, (shown as the surface feature in Figure 3d). By applying a 2-dimensional corrective ratio in this fashion, we adjust the model's SW-NE trend.

The adjusted MWL to datum height is shown in Figure 4a, and comparison to observations ($LLWLT_o$-$LLWLT_m$) is shown in Figure 4b. The differences between the adjusted $LLWLT_m$ and observations are normally distributed (inset of Figure 4b), with a standard deviation of ~ 10 cm. Figure 4b indicates that $LLWLT_m$ is least accurate in areas where the datum slopes steeply, such as in the St. Lawrence Estuary and the Northumberland Straight, and at the mouths of rivers, two areas where large-scale ocean circulation models such as G5 are expected to be limited by their resolution.


a) Adjusted model datum (LLWLTm) and observed target datum (LLWLTo)

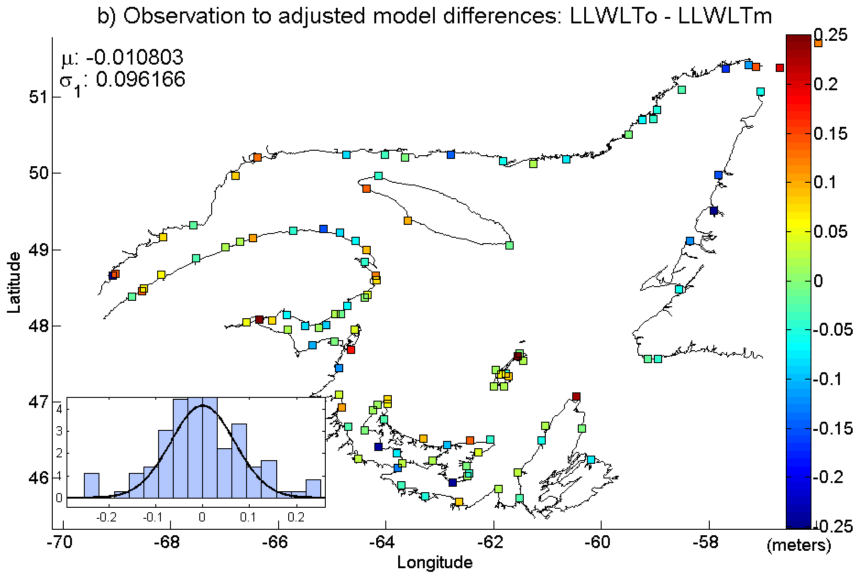

b) Observation to adjusted model differences: LLWLTo - LLWLTm

Figure 4.* a) Adjusted *$LLWLT_m$*; b) Differences between *$LLWLT_o$* and adjusted *$LLWLT_m$

3.2 Geoid tie to NAD83(CSRS)

CGG2010 is a gravitational equipotential surface whose potential (in m^2/s^2) is chosen to best fit global mean sea level in a least-squared sense (NRCan, 2012). A better approximation of MSL for our purposes can be made by shifting the geoid to fit MWL over the model domain. For a shift of this magnitude (a few decimetres), the change in shape of the geoid itself is negligible (M. Véronneau, personal communication), and therefore we calculate a uniform vertical shift directly from our observations at each gauge as:

$$dN = (h + Z_0) - N \tag{3}$$

We then take an average of *dN* over all GPS-observed gauges, and apply this value as a correction term for *N* at model nodes, as in equation (1). As seen in Figure 5, the distribution of *dN* does not appear to have any trends, and therefore a shift applied everywhere equally is appropriate. For this set of gauges, *dN* is normally distributed, with $\langle dN \rangle \cong -0.45\,\text{m}$. When several model domains are joined, this procedure will have to be optimized; for instance, when joining the St. Lawrence model to the Gulf model presented here, it may be appropriate to use $dN \cong -0.33\,\text{m}$ everywhere, the theoretical value of *dN* at Rimouski where the two models will meet.

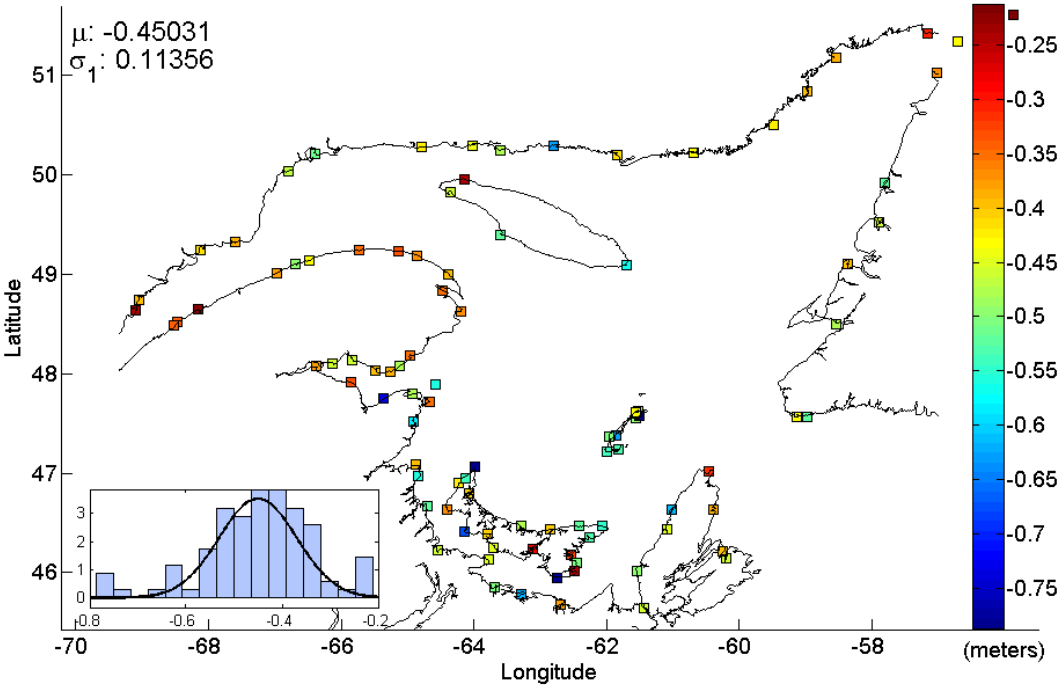
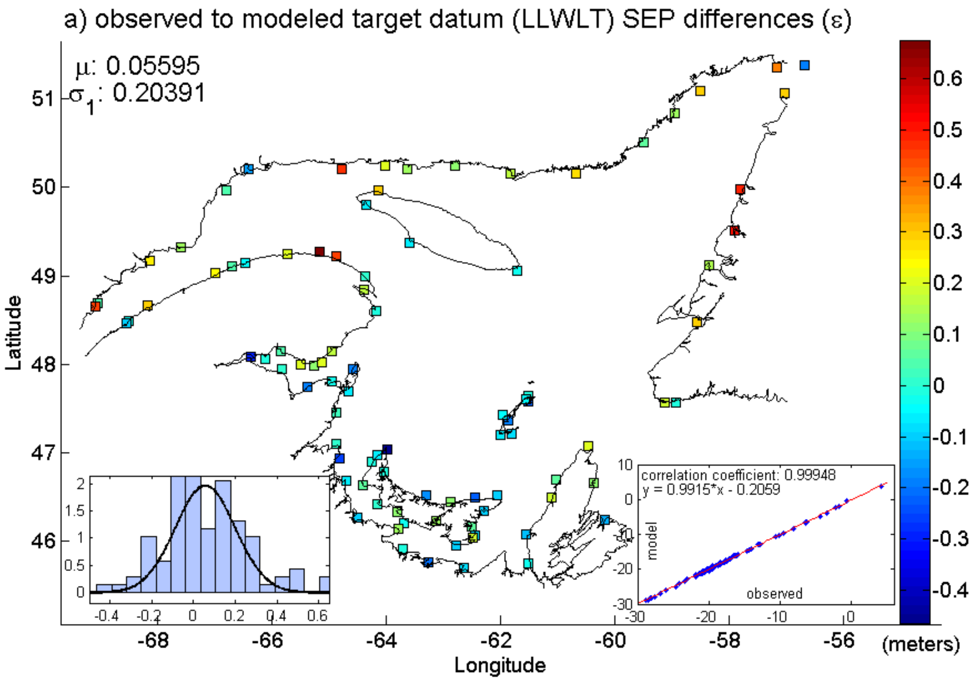


***Figure 5*. *dN* at selected gauges.**

The model SEP is then calculated using Equation 1, using the adjusted $LLWLT_m$ and <*dN*>. The resulting difference between the observed and calculated LLWLT-ellipsoid separation (ε from Equation 2) is shown in Figure 6a. The slightly bi-modal distribution of the discrepancies appears to stem from the DOT_m, although in theory it should have the opposite effect as it brings the model's surface closer to an equipotential, and therefore more parallel to the geoid. This issue is currently being investigated. Nevertheless, the model correlates well with observations (correlation coefficient > .99), with a standard deviation of ~ 20 cm. The model bias of ~5.6 cm is removed from the model SEP, shown in Figure 6b, reducing the uncertainty of the model SEP to $\varepsilon \approx \pm 20\,\text{cm}$.


a) observed to modeled target datum (LLWLT) SEP differences (ε)
μ: 0.05595
σ1: 0.20391
Latitude
Longitude
51
50
49
48
47
46
-68
-66
-64
-62
-60
-58
-56
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
(meters)
2
1.5
1
0.5
0
-0.4
-0.2
0
0.2
0.4
0.6
correlation coefficient: 0.99948
y = 0.9915*x - 0.2059
model
observed
10
0
-10
-20
-30
-30
-20
-10
0

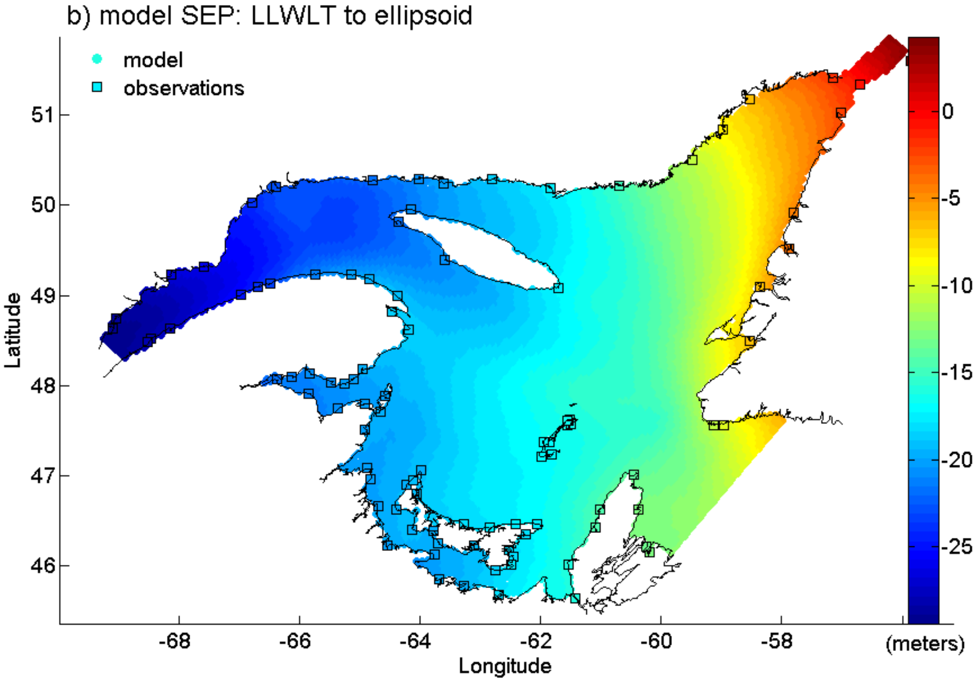


***Figure 6*. a) Differences between observed and modeled target datums; b) model SEP.**

The model SEP is dominated by an east-west trend stemming from the large wavelength geoid undulation. Smaller wavelength features are due to variations in the model's tidal regime and dynamic ocean topography, and smaller-scale variations in gravity due to local geology captured by the CGG2010.

3.3 'Warping'

A number of approaches are available to warp the model SEP to honour official CD values whose ellipsoid height is known. One is to calculate the difference between the modeled datum and CD at each tide station ($\varepsilon + \Delta$), interpolate these values over the model domain and apply the result as a mask to the model SEP. Even if care is taken not to let the interpolation reach over land, this approach has two main limitations, particularly over large bodies of water where tide stations are sparse. First, variations in Δ from one station to another are not correlated and moreover have little physical significance; the same is true for most of the errors that contribute to ε. Therefore, there is no reason to impose these 'errors' to the whole of the model domain. Second, interpolation by definition will reach from one shore to the next; in channels and small bays this may be appropriate for systematic biases, for our study area it is not.

Instead, we isolate the boundary nodes of the model and constrain them to CD at gauges, which now define a model shoreline. We calculate $\varepsilon + \Delta$ at each gauge, which is then propagated to all nodes of the model shoreline by inverse-distance weighting to the two closest gauges, one in each direction along the coast. For the Gulf, we separate the shoreline in 4 parts corresponding to each of the 4 main coastlines (Anticosti, PEI, the Magdalen Islands and the remainder of the boundary), so that gauges on any of the major islands only influenced the model shoreline around that island. The result, shown in

Figure 7a, is then applied as a mask to the model SEP values on its shoreline. At gauges CD is exactly reproduced; between gauges the model SEP is shifted and tilted linearly in the vertical direction but retains the model's shape, as shown in the profile in Figure 7b. This now acts as a control shoreline (CS), from which we propagate values offshore.

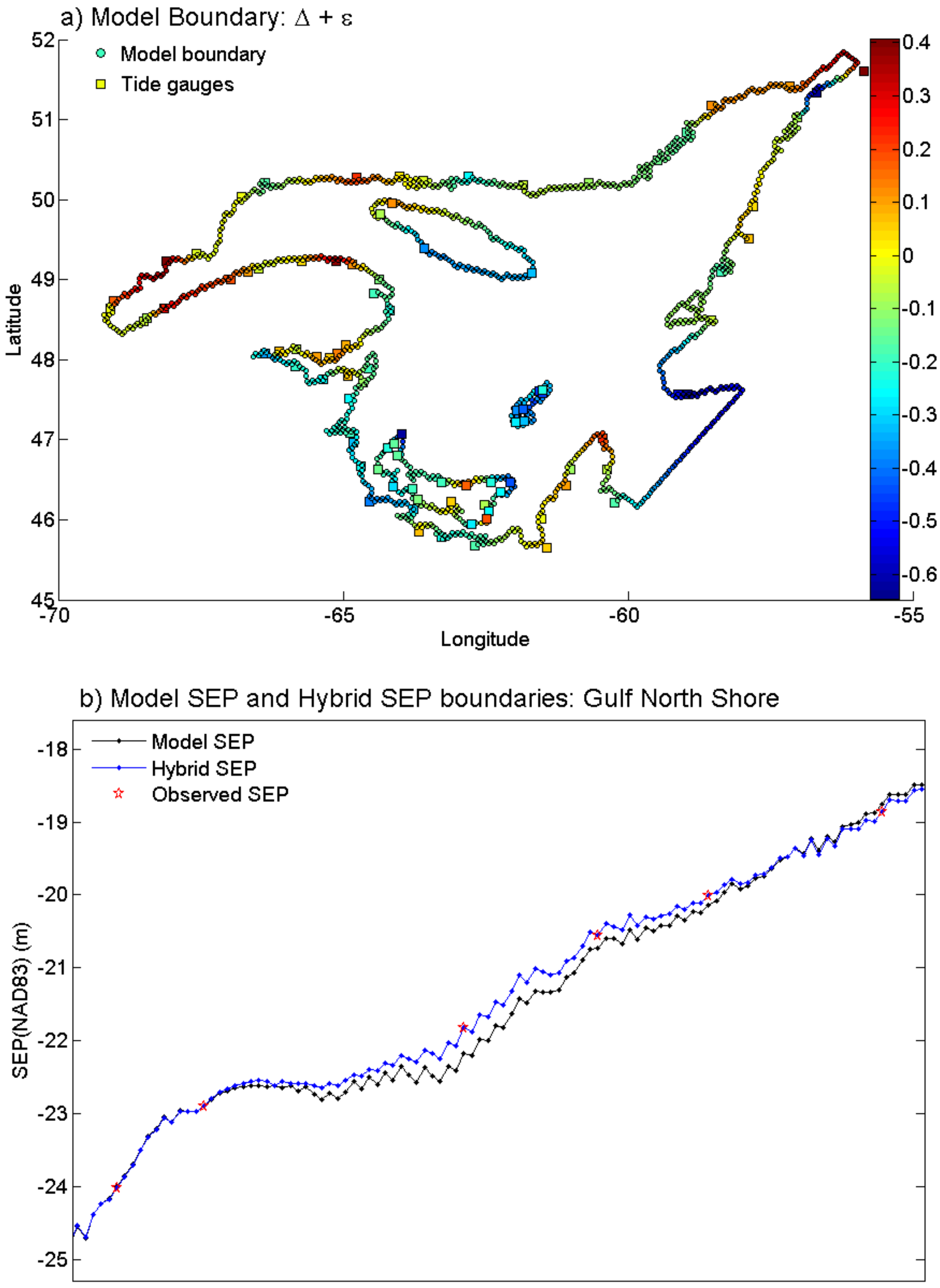


***Figure 7.* Note that in b) the zig-zag shape of the outline is because in places where the boundary is 2 nodes thick (where the boundary is close to a 45° angle to the model grid, or where there are small islands), each point may have a slightly different SEP value.**

3.4 Control Shoreline Propagation Offshore

The control shoreline will define CD at the boundary of the hybrid SEP, while the model SEP will define CD in its interior. Joining the control shoreline to the interior will be a transition zone (TZ) where values from the CS transition to pure model values over some distance. The 3 zones are shown in Figure 8.

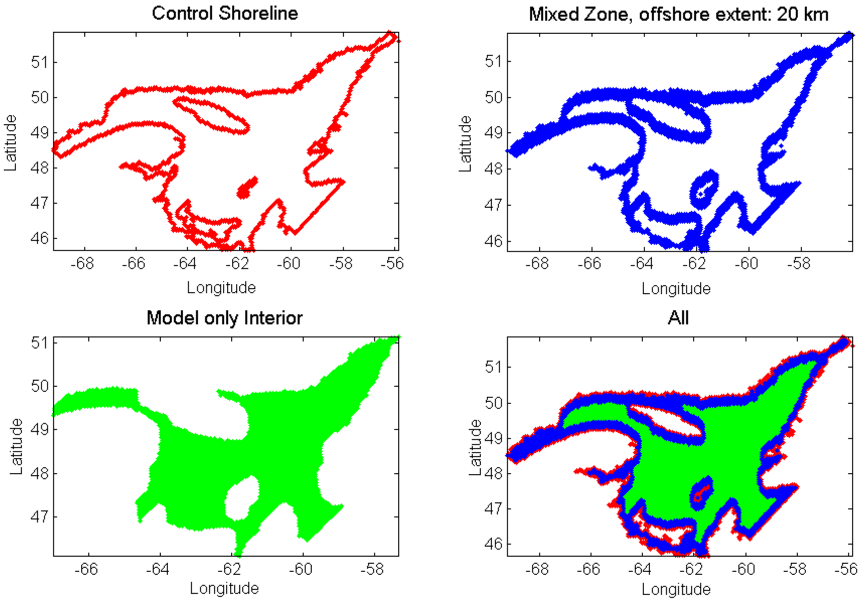


***Figure 8*. Zones of the hybrid SEP.**

A number of procedures can be imagined to move from warped shoreline values to pure model values in the TZ. For instance, one could linearly exchange CS values to model SEP values over a distance *x*, or using a smoothed step function from one to the other at an appropriate bathymetric contour. The hybrid SEP will likely require a combination of different approaches to account for complex shorelines, narrow passages and busy navigation zones. Here we show results for the simplest (and easiest to visualize) approach, that is, we linearly transition from CS to model SEP values across a 20 km zone.

The TZ calculation is illustrated in Figure 9. In this case, an inverse distance weighted average of the 10 closest points in CS is calculated for each point in the TZ; in this example the TZ point is plotted as a star, and the 10 CS points from which it is averaged it are circled. Care must be taken that the inverse distance weighting does not reach across important islands, especially in areas like the Magdalen Islands where the tidal regime is differs significantly on each side of the island and where there is a high density of observations. We do, however, want it to reach across straights and channels. This is most easily achieved by limiting the number of CS points used in the weighted average.

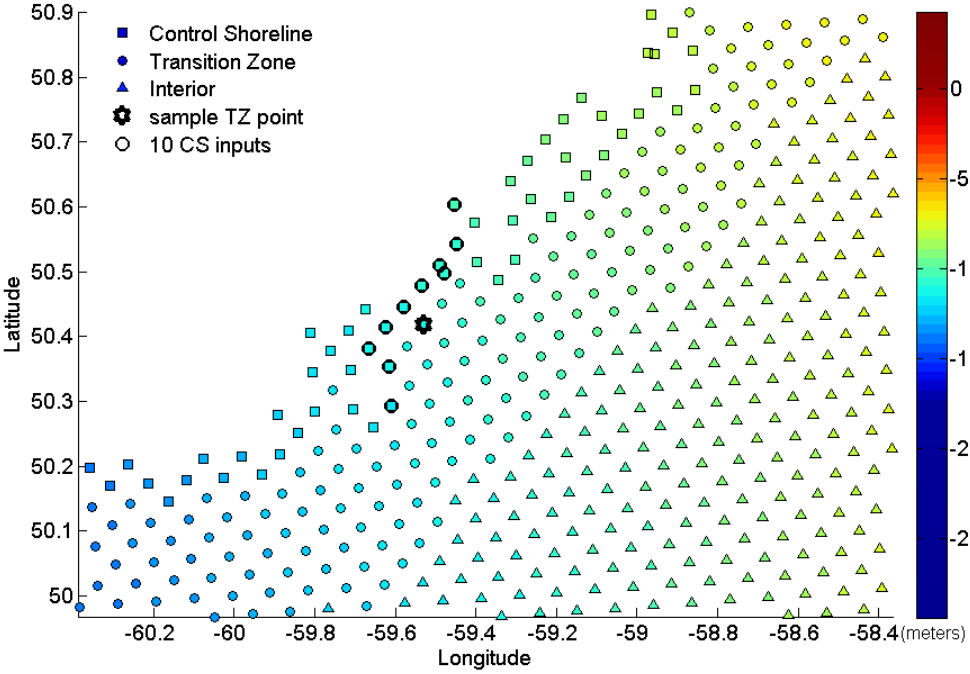


Figure 9. **Close-up of SEP zones and illustration of transition zone weighting scheme.**

The weighted average of the CS on the nodes of the TZ is then transitioned so that

$$SEP_{TZ} = (1-R)*SEP_{WCS} + R*SEP_m \tag{4}$$

$$R = x/D \tag{5}$$

where x is the distance from the boundary of the model, D is the width of the TZ, SEP_{WCS} is the weighted average of the Control Shoreline, and SEP_m is the model SEP as defined in equation (2).

The hybrid SEP is an amalgamation of the Control Shoreline, the TZ and the Interior, and looks qualitatively very similar to the model SEP of Figure 6b. A more useful visualization of it and the input surfaces a cross- section, shown in Figure 10 for a section running from the top of the Gaspe Peninsula to the north shore of the St. Lawrence Estuary (see Figure 10 inset). This area was chosen as there is a bowl-shaped feature in the geoid which dominates the SEP in this area (*N*, in green), making the various surfaces easy to distinguish. The model SEP (red) indicates that the lower water datum is ~ 2 m below CGG2010 here; it is not quite parallel to the geoid, due to both the DOT (higher on the south shore of the Gulf), and to varying tidal regimes. Finally, the final Gulf SEP (black) is equal to the Control Shoreline (pink) at the shore, and to the model SEP in the center. In the TZ, it transitions linearly from the control shoreline extended 20 km offshore (TZ(w) - yellow) and the model SEP. This case is particular because the geoid dips steeply immediately offshore; in most other areas, the hybrid and model SEP's are not as divergent in the TZ.

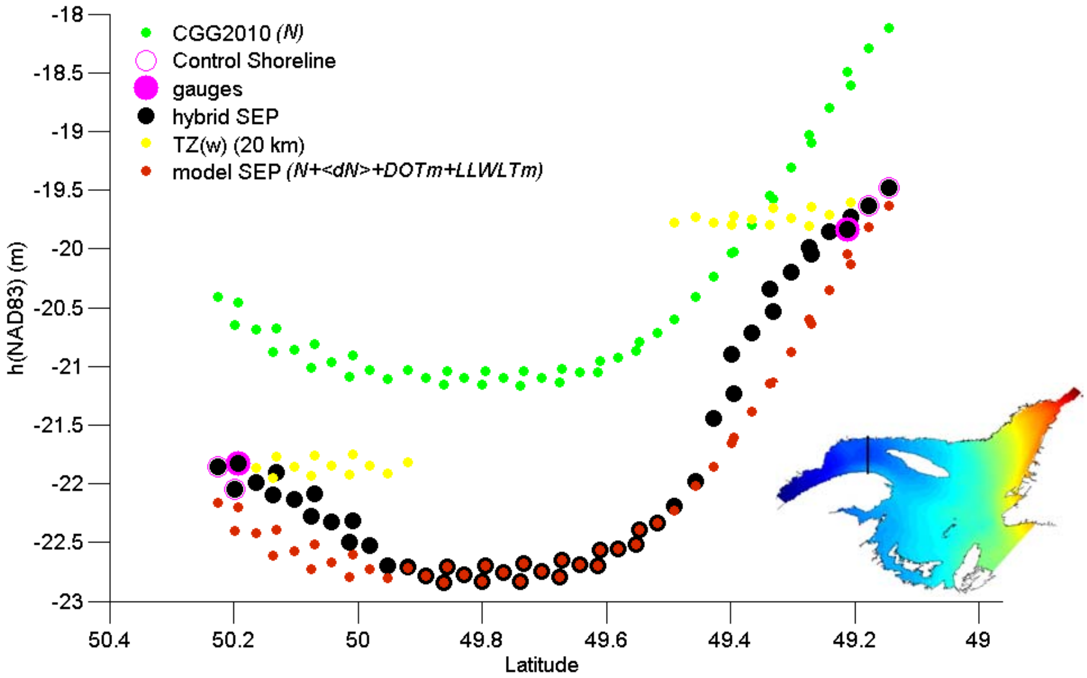


***Figure 10*. Vertical cross-section of the SEP and its input surfaces along the line shown in the inset map.**

4. CONCLUSION

We have outlined a method for calculating a seamless chart datum surface using a geoid and an oceanic circulation model to connect isolated observations of CD to NAD83 separation at tide gauges, in a manner that minimizes off-grid interpolations and takes into account shoreline geometry. Observations and model data for the Gulf of St. Lawrence were used to illustrate the process. The discrepancy between observations and modeled target datums (the model SEP) for the Gulf in the calculation shown here is ~ 5.6 ± 20 cm (mean + standard deviation); the model bias of 5.6 cm is removed in the hybrid SEP, which is then forced to fit established values of CD-NAD83.

This protocol for building a SEP and its error model are still under development. In addition, field campaigns are ongoing. As a result, the model will be recalculated regularly. Some of the calculation may be automated; however, the nature of the inputs (complex shorelines, multiple types and quality of data, etc) means that each calculation will require informed selection and inspection of input data and final values at a local scale, using visual inspection and the experience of each region's tidal officers. Thus the modeling procedure and the input data will be refined, and the accuracy will improve, providing an invaluable tool for hydrography, navigation, and a variety of scientific endeavours.

REFERENCES

Backhaus, J.O., 1983. A semi-implicit scheme for the shallow water equations for application to shelf sea modeling. *Cont. Shelf. Res.* **2**: 234–254.

Backhaus, J.O., 1985. A three-dimensional model for the simulation of shelf-sea dynamics. *Dt. Hydtrogr. Z.* **38**: 165–187.

de Lange Boom, B., 2012. Vertical Datum Separation Models for the British Columbia Coast, *CHC 2012 Conference Proceedings,* Niagara Falls, ON, Canada.

Foreman, M.G.G., 1977. Manual for Tidal Heights Analysis and Prediction. *Pacific Marine Science Report 77-10,* Institute of Ocean Sciences, Patricia Bay, Sidney, B.C., 58 pp. (2004 revision).

Godin, G. G., 1972. *The Analysis of Tides*. University of Toronto Press, 264 pp.

Lefaivre, D., A. Godin, D. Dodd, T. Herron, P. MacAulay, D. Sinnott, 2010. The Continuous Vertical Datum Canadian Waters Project (Beginnings, Vision, Methods and Progress), *CHC 2010 Conference Proceedings,* Quebec, QC, Canada.

NRCan, 2011. GPS-H, *Natural Resources Canada, Earth Sciences Sector, Canadian Spatial Reference System*, http://www.geod.rncan.gc.ca/tools-outils/gpsh_e.php, accessed October 24, 2011.

O'Reilly, C.S., Parsons & D. Langelier, 1996. A Seamless Vertical Reference Surface for Hydrographic Data Acquisition and Information Management, *CHC 1996,* June.

Saucier, F.J., F. Roy, D. Gilbert, P. Pellerin et H. Ritchie, 2003. Modeling the formation and circulation processes of water masses and sea ice in the Gulf of St. Lawrence, *J. Geophys. Res.*, 108(C8): 3269-3289.

Senneville, S. & S. St-Onge Drouin, 2010. Rapport de validation : Simulation du golfe du Saint-Laurent 1997-2007 pour le MPO, 65 pp.

Stronach, J. A., J. Backhaus and T. S. Murty, 1993. An update on the numerical simulation of oceanographic processes in the waters between Vancouver Island and the mainland: the GF8 model. Oceanography and Marine Biology: An annual Review 31, 1-86.

NRCan, 2012. Height Reference System Modernization, Geoid Modelling - Public Models, *Natural Resources Canada, Earth Sciences Sector, Canadian Spatial Reference System*, http://www.geod.nrcan.gc.ca/hm/public_model_e.php, accessed April 10, 2012.

BIOGRAPHICAL NOTE

Catherine Robin received her doctorate in Physics, Geology & Environmental Science from the University of Toronto in 2010. She currently holds an NSERC Visiting Fellowship with Natural Resources Canada and the Department of Fisheries and Oceans.

CONTACTS

Dr. Catherine Robin
Natural Resources Canada/Department of Fisheries and Oceans
615 Booth St.,
Ottawa, ONTARIO
Tel. +1-613-992-4175
Fax +1-613-995-3215
Email: crobin@nrcan.gc.ca